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Spin-polarized one-dimensional electron gas with short-range interaction: analytical results

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Abstract. We study spin-polarization effects in the one-dimensional electron gas with parabolic dispersion and a repulsive delta-function interaction potential. Using quasi-exact results for the unpolarized system and exact results for the fully polarized system we derive quasi-exact *analytical results* for the ground-state energy as a function of the spin polarization and the interaction strength. The spin susceptibility as a function of the interaction strength is calculated and compared with other calculations. We present results for the sound velocity of collective spin excitations and the sound velocity of collective density excitations.

1. Introduction

In this paper we calculate the spin susceptibility of a one-dimensional electron gas with a repulsive short-range interaction potential. Such a model was discussed long ago [1, 2] and the exact ground-state energy (GSE) of this model has been studied [3]. The GSE was also calculated within the ladder approximation [4] and was found to be in reasonable agreement with the exact result. Analytical results for the GSE, which are exact for weak and strong coupling, have been given recently within the ladder approach [5]. Spin-dependent properties of this model in the strong-coupling limit have not been discussed in the literature. However, it is known that the fully polarized system is a non-interacting system due to the action of the Pauli principle and the short-range nature of the interaction [6]. In this paper we present an analytical formula for the GSE, including polarization effects, which is exact for small and large coupling. This GSE can be used to calculate the spin susceptibility and the collective spin-density modes.

Many-body effects in interacting quantum liquids can also be studied within the Singwi–Tosi–Land–Sjölander (STLS) approach [7] (compressibility) and the Lobo–Singwi–Tosi (LST) approach [8] (spin susceptibility). A local-field correction describes many-body effects within these two approaches [9]. We have recently applied the STLS and the LST approach to the one-dimensional electron gas with a short-range interaction [10]. In this paper we will argue that the GSE calculation of the STLS approach, generalized to polarized systems, is a good approach to describe many-body effects for not too large coupling.

We will also show in this paper that no Bloch instability, a transition from an unpolarized to a polarized system [11], occurs within our short-range interaction model. This is in

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agreement with a theorem that a polarized system always has a larger GSE than a non-polarized system [12]. On the other hand we argued before [13] that for a long-range interaction potential such an instability occurs and we calculated the spin susceptibility for that model within a generalized STLS approach [14]. It is, therefore, important to get information concerning the validity range of the STLS approach. In this paper we compare the results of the STLS approach with quasi-exact results. Our detailed calculation of the STLS approach will be published elsewhere [15].

The paper is organized as follows. In section 2 we present the model. The analytical results for the ground-state energy are described in section 3. The spin susceptibility is given in section 4. In section 5 we discuss the local-field correction. An extensive discussion of our results is given in section 6. We conclude in section 7.

2. Model

We study a one-dimensional electron gas with kinetic energy, characterized by an effective mass m and the parabolic dispersion $\varepsilon(\boldsymbol{q}) = q^2/2m$. We use for the Planck constant $h/2\pi = 1$. The interaction is characterized by the potential V_0. The interaction potential between two particles at r_1 and r_2 is given by $V(r_1, r_2) = V_0\delta(r_1 - r_2)$. In the Fourier space the interaction potential is independent of the wave number q and expressed as $V(q) = V_0 > 0$. The electron density N, the electron mass and the strength of the interaction define the dimensionless parameter γ as $\gamma = mV_0/N = \pi V_0/2v_F > 0$ and $v_F = k_F/m$ is the Fermi velocity. The electron density defines the Fermi wave number k_F via $N = 2k_F/\pi$. $\rho_F = N/2\varepsilon_F$ is the density of states at the Fermi energy ε_F of the non-interacting electron gas.

The spin-polarization parameter ξ with $0 \leqslant |\xi| \leqslant 1$ is given by $\xi = (N_+ - N_-)/N$ with $N = N_+ + N_-$. The electron densities $N_\pm$ are expressed as $N_\pm = N(1 \pm |\xi|)/2$. The Fermi wave numbers of the polarized subsystems are written as $k_{F\pm} = k_F(1 \pm |\xi|) = \pi N_\pm$. The essential parameters of the theory are N and ξ for the electron gas and γ for the interaction.

Weak coupling means $\gamma < 1$ and strong coupling is represented by $\gamma > 10$. Of course, it is the intermediate regime $1 < \gamma < 10$ where our analytical results are most important for experimenters. In the weak coupling regime the physics is determined by exchange effects, which can be calculated analytically. In fact, we we will show that the STLS approach is a good theory for $\gamma < 10$.

3. Ground-state energy

The ground-state energy $\varepsilon_g(\gamma, \xi)$ per particle is related to the total energy $E_g(\gamma, \xi) = N\varepsilon_g(\gamma, \xi)$. In the following we present results for $\varepsilon_g(\gamma, \xi)$ in units of $N^2/2m$ via

$$\varepsilon_g(\gamma, \xi) = N^2\varepsilon_0(\gamma, \xi)/2m \tag{1a}$$

and give explicit expressions for $\varepsilon_0(\gamma, \xi)$, which is a dimensionless quantity. $\varepsilon_0(\gamma, \xi)$ can be written as [16]

$$\varepsilon_0(\gamma, \xi) = \varepsilon_{kin}(\gamma, \xi) + \varepsilon_H(\gamma, \xi) + \varepsilon_{ex}(\gamma, \xi) + \varepsilon_{cor}(\gamma, \xi) \tag{1b}$$

with the kinetic (kin) energy contribution as

$$\varepsilon_{kin}(\gamma, \xi) = \pi^2(1 + 3\xi^2)/12 \tag{2a}$$

the Hartree (H) energy contribution as

$$\varepsilon_H(\gamma, \xi) = \gamma \tag{2b}$$

and the exchange (ex) energy contribution as

$$\varepsilon_{ex}(\gamma,\xi) = -\gamma(1+\xi^2)/2. \tag{2c}$$

We note that within the Hartree–Fock approximation (HFA) the interaction energy is given by $\varepsilon_{HFA}(\gamma,\xi) = \varepsilon_H(\gamma,\xi) + \varepsilon_{ex}(\gamma,\xi) = \gamma(1-\xi^2)/2$.

In a recent paper [5] we derived, using the ladder theory, for the correlation energy $\varepsilon_{cor}(\gamma,\xi=0) = -\gamma^2/[\pi^2(1+2\gamma/\pi^2)]$. Including the spin polarization we use the *ansatz* $\varepsilon_{cor}(\gamma,\xi) = \varepsilon_{cor}(\gamma,\xi=0)f(\xi)$. $f(\xi)$ describes polarization effects with $f(\xi=0)=1$. What more can we say about $f(\xi)$? Interaction effects are absent for the fully polarized system: an electron system with all spins aligned cannot interact via a short-range potential. It follows that $\varepsilon_0(\gamma,\xi=\pm1) = \pi^2/3$ and we conclude that $f(\xi=\pm1)=0$. We note that $f(\xi)$ is an even function of ξ. In the strong coupling limit it is known that $\varepsilon_0(\gamma\to\infty,\xi=0) = \varepsilon_0(\gamma\to\infty,\xi=\pm1) = \pi^2/3$. It is obvious that this holds for every polarization: $\varepsilon_0(\gamma\to\infty,-1\leqslant\xi\leqslant1) = \pi^2/3$. From our results for $\gamma\to\infty$ we obtain $\gamma^1[1-\xi^2-f(\xi)]=0$ and $\gamma^0[1+3\xi^2+3f(\xi)]=4$: both equations are solved by $f(\xi)=1-\xi^2$. This form of $f(\xi)$ is in agreement with the polarization dependence of the interaction energy obtained within the HFA.

For the three-dimensional electron gas with long-range Coulomb interaction it was proposed that polarization effects are described by $\varepsilon_{cor}(r_s,\xi) = \varepsilon_{cor}(r_s,\xi=0) + [\varepsilon_{cor}(r_s,\xi=\pm1) - \varepsilon_{cor}(r_s,\xi=0)]h(\xi)$ with $h(\xi=0)=0$ and $h(\xi=\pm1)=1$ [17]. r_s is the Wigner–Seitz parameter. $h(\xi)$ is proportional to the factor describing polarization effects of the exchange term, which is in our case $(1+\xi^2)$, see equation (2c): $h(\xi) = C_1 + C_2(1+\xi^2)$. In order to satisfy $h(\xi=0)=0$ and $h(\xi=\pm1)=1$ we obtain $C_2 = 1 = -C_1$ and we conclude that for our model $h(\xi)=\xi^2$. With $\varepsilon_{cor}(\gamma,\xi=\pm1)=0$ we derive $\varepsilon_{cor}(\gamma,\xi) = \varepsilon_{cor}(\gamma,\xi=0) - \varepsilon_{cor}(\gamma,\xi=0)h(\xi)$ and we find $\varepsilon_{cor}(\gamma,\xi) = \varepsilon_{cor}(\gamma,\xi=0)f(\xi)$ with $f(\xi) = 1-h(\xi) = 1-\xi^2$.

We conclude that the correlation energy, including polarization effects, is given by

$$\varepsilon_{cor}(\gamma,\xi) = -\frac{\gamma^2(1-\xi^2)}{\pi^2(1+2\gamma/\pi^2)}. \tag{3}$$

For weak coupling the energy difference $\Delta\varepsilon_{cor}(\gamma,\xi\to0) = \varepsilon_{cor}(\gamma,\xi\to0) - \varepsilon_{cor}(\gamma,\xi=0)$ is given as $\Delta\varepsilon_{cor}(\gamma,\xi\to0) = \gamma^2\xi^2/\pi^2$. Using the mean spherical approximation (MSA) [14], which is exact in the weak coupling limit, one obtains $\Delta\varepsilon_{cor,MSA}(\gamma,\xi\to0) = 3\gamma^2\xi^2/2\pi^2$ for $\gamma\to0$. The MSA represents the random-phase approximation (RPA) or the mean-field approximation. However, in the MSA the static structure factor is used in analytical form. The difference in $\Delta\varepsilon_{cor}(\gamma,\xi\to0)$ (the factor 1 instead of 3/2) in the weak coupling regime is the price we have to pay for using a simple function for $f(\xi)$: remember that $f(\xi)=1-\xi^2$ was determined from strong coupling (in section 6 we come back to this point). However, from our numerical results, see figure 1, it is clear that the difference is of minor importance because for weak coupling the exchange term dominates the GSE.

With $f(\xi)=1-\xi^2$ we get for the GSE

$$\varepsilon_0(\gamma,\xi) = \frac{\pi^2(1+3\xi^2)}{12} + \frac{\gamma(1-\xi^2)}{2(1+2\gamma/\pi^2)}. \tag{4}$$

This is the essential equation of our paper. For $\xi=0$ this function was given before and the compressibility was calculated and related to critical exponents [5]. In the following we derive conclusions by using the GSE as given in equation (4). The GSE versus γ is shown in figure 1 for different values of the spin-polarization parameter ξ together with results obtained within the STLS approach [15].

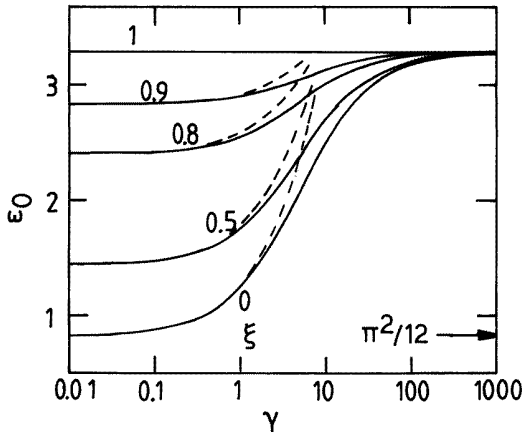

Figure 1. Ground-state energy ε_0 versus interaction strength parameter γ according to equation (4) for different values of the spin-polarization parameter ξ. The dashed lines represent the results within the STLS approach [15].

The total energy also can be written as

$$\varepsilon_0(\gamma,\,\xi) = \frac{\pi^2}{12}\left(1+3\xi^2\right) + \int_0^\gamma \mathrm{d}\lambda g(\lambda,\,\xi,\,z=0) \tag{5}$$

and $g(\gamma,\xi,z=0)$ is the pair distribution function. We conclude that $g(\gamma,\xi,z=0)$ is given by

$$g(\gamma,\,\xi,\,z=0) = \left(1-\xi^2\right)\Big/\left[2\left(1+2\gamma/\pi^2\right)^2\right]. \tag{6}$$

Due to the Pauli principle the pair distribution function vanishes for the polarized system with $\xi = \pm 1$.

With the ground-state energy one can calculate the chemical potential μ as

$$\mu/\varepsilon_F = 1+3\xi^2+4\gamma\left(1+3\gamma/\pi^2\right)\left(1-\xi^2\right)\Big/\left[\pi^2\left(1+2\gamma/\pi^2\right)^2\right] \tag{7a}$$

the kinetic energy t per particle as

$$t/\varepsilon_F = \left[1+3\xi^2+12\gamma^2\left(1-\xi^2\right)\Big/\left[\pi^4\left(1+2\gamma/\pi^2\right)^2\right]\right]/3 \tag{7b}$$

and the potential energy v per particle as

$$v/\varepsilon_F = 2\gamma\left(1-\xi^2\right)\Big/\left[\pi^2\left(1+2\gamma/\pi^2\right)^2\right]. \tag{7c}$$

Note that $v=0$ for $\gamma=0$, $1/\gamma=0$ or $\xi=\pm1$. For $1/\gamma=0$ interaction effects disappear, $v=0$, and the particles behave as free (spinless) particles [6]. Our results for μ, t and v versus γ are shown in figure 2 for different values of the polarization parameter. We note that the potential energy v has a maximum at $\gamma=\gamma_{HFA}=\pi^2/2\approx 4.9$ and v decreases with increasing polarization.

4. Spin susceptibility

The spin susceptibility κ_s can be expressed by the second derivative of the GSE as $\partial^2 E_g/N^2\partial\xi^2 = \pi v_F\kappa_0/2\kappa_s$ [16] with $\kappa_0 = 4m/\pi^2N^3$ as the spin susceptibility of the

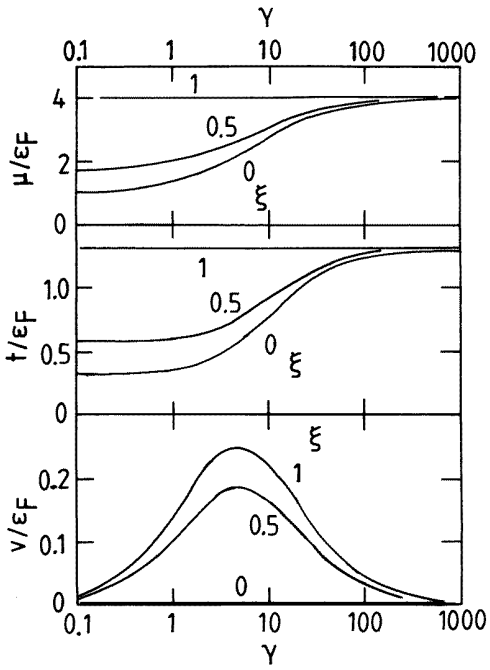

Figure 2. Chemical potential μ, kinetic energy t and potential energy v per particle (in units of the Fermi energy ε_F of the free electron gas) versus interaction strength parameter γ for different values of the spin-polarization parameter ξ according to equation (7).

free electron gas. One obtains

$$\frac{\kappa_0}{\kappa_s} = 1 + \frac{2}{\pi^2} \lim_{\zeta \to 0} \left[\frac{\partial^2 (\varepsilon_{ex} + \varepsilon_{cor})}{\partial \xi^2} \right] = 1 + \frac{\kappa_0}{\kappa_{s,ex}} + \frac{2}{\pi^2} \alpha_s. \tag{8}$$

$\alpha_s = \lim_{\xi \to 0} [\partial^2 \varepsilon_{cor}(\gamma, \xi)/\partial \xi^2]$ is called the spin stiffness. We mention that the Hartree term does not contribute to the spin susceptibility. The exchange contribution is $\kappa_0/\kappa_{s,ex} = -2\gamma/\pi^2$. With equation (4) we derive for the correlation contribution $\alpha_s = 2\gamma^2/[\pi^2(1 + 2\gamma/\pi^2)]$. The limiting behaviour is $\alpha_s = 0.203\gamma^2$ for $\gamma \to 0$ and $\alpha_s = \gamma(1 - \pi^2/2\gamma)$ for $\gamma \to \infty$.

With equation (4) we find the analytical result for the spin susceptibility as

$$\frac{\kappa_0}{\kappa_s} = \frac{1}{1 + 2\gamma/\pi^2}. \tag{9}$$

The asymptotic result is written as $\kappa_0/\kappa_s = 1 - 2\gamma/\pi^2 + 4\gamma^2/\pi^4$ for $\gamma \to 0$. The term $-2\gamma/\pi^2$ represents the exchange correction. The term $4\gamma^2/\pi^4$ is the first correlation correction to the inverse spin susceptibility. In the strong coupling limit we obtain $\kappa_0/\kappa_s = \pi^2(1 - \pi^2/2\gamma)/2\gamma$ for $\gamma \to \infty$. We mention that $\kappa_s/\kappa_0 = 1 + 2\gamma/\pi^2$ increases linearly with increasing coupling, a behaviour also found for the Hubbard model [18].

It is interesting to note that the spin susceptibility, as given in equation (9), can also be obtained from $\kappa_0/\kappa_s = 4[\varepsilon_0(\gamma, \xi = 1) - \varepsilon_0(\gamma, \xi = 0)]/\pi^2$. A similar equation is sometimes used for interacting systems with a long-range Coulomb interaction. Within three different methods, (i) $f(\xi) = 1 - \xi^2$, (ii) $h(\xi) = \xi^2$ and (iii) $\varepsilon_0(\gamma, \xi = 1) - \varepsilon_0(\gamma, \xi = 0)$, we found κ_0/κ_s as given in equation (9). We believe that this fact gives credit to our expression for the spin susceptibility.

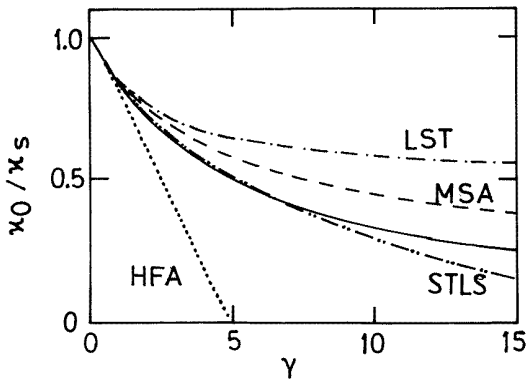

Figure 3. Inverse spin susceptibility $1/\kappa_s$ (in units of the inverse spin susceptibility of the free electron gas $1/\kappa_0$) versus interaction strength parameter γ according to equation (9) as the solid line. The results according to the HFA, the MSA, the LST approach [10] and the STLS approach [15] are also shown.

Let us now discuss some approximating expressions for the spin susceptibility. Within the HFA, neglecting correlation, one obtains

$$\frac{\kappa_0}{\kappa_{s,HFA}} = 1 - 2\gamma/\pi^2. \tag{10}$$

In a recent paper an analytical result for the spin susceptibility of a one-dimensional electron gas with a long-range interaction potential was calculated within the MSA [14]. For the present model we obtain

$$\frac{\kappa_0}{\kappa_{s,MSA}} = \frac{1}{(1+4\gamma/\pi^2)^{1/2}}. \tag{11}$$

The weak coupling expansion is written as $\kappa_0/\kappa_{s,MSA} = 1 - 2\gamma/\pi^2 + 6\gamma^2/\pi^4$ for $\gamma \to 0$. We note that the correlation correction is $6\gamma^2/\pi^4$ and somewhat larger that found in our analytical approach. Numerically, this difference is barely visible, see figure 3.

Our different results for the spin susceptibility versus γ are shown in figure 3. We find $1/\kappa_{s,HFA} < 1/\kappa_s < 1/\kappa_{s,MSA}$. We note that $\kappa_0/\kappa_s > 0$ and never becomes zero. This shows that the non-polarized system is stable for any value of the interaction parameter γ. A Bloch instability $\kappa_0/\kappa_{s,HFA} = 0$ occurs within the HFA at $\gamma_{HFA} = \pi^2/2$. Correlation effects shift this instability point to $\gamma \to \infty$.

Our calculation using the STLS approach, also shown in figure 3, indicates that the STLS approach has a validity range of about $\gamma < 10 \approx 2\gamma_{HFA}$. The term STLS is applied for the calculation of the spin susceptibility using GSE calculations [15]. Within the LST approach, see our results in [10], the spin susceptibility is calculated via the local-field correction and this approach is rather inaccurate, as can be seen in figure 3. On the other hand we mention that the LST approach does not predict an (artificial) instability as does the HFA.

5. Local-field correction and collective modes

Our results for κ_0/κ_s can be applied to obtain information on the dynamic properties of the interacting electron gas. We define the local-field correction $G_s(q,\omega)$ by the dynamic

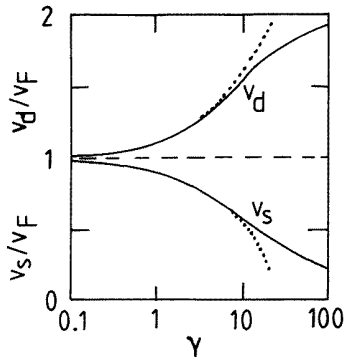

Figure 4. Density velocity v_d and spin velocity v_s (in units of the Fermi velocity v_F of the free electron gas) versus interaction strength parameter γ. The dotted lines correspond to the STLS approach derived from GSE calculations [15].

spin-density response function $X_s(q, \omega)$ as [9]

$$X_s(q, \omega) = \frac{X_0(q, \omega)}{1 - G_s(q, \omega) X_0(q, \omega)}. \tag{12}$$

$X_0(q, \omega)$ is the Lindhard function of the free electron gas. Note that the local-field correction depends on q and ω. By using the compressibility sum rule $X_s(q \to 0, \omega = 0) = N^2 \kappa_s$ one finds [16]

$$\kappa_0/\kappa_s = 1 - 4\gamma G_s(0, 0)/\pi^2. \tag{13}$$

With the analytical expression for κ_0/κ_s in equation (9) we derive

$$G_s(0, 0) = 1/[2(1 + 2\gamma/\pi^2)] \tag{14}$$

with $G_s(0, 0) = (1 - 2\gamma/\pi^2 + 4\gamma^2/\pi^4)/2$ for $\gamma \to 0$ and $G_s(0, 0) = \pi^2/4\gamma$ for $\gamma \to \infty$. Within the LST approach, where $G_s(q, \omega) = G_s$ is independent of ω and q, we found $G_s = \pi^2/8\gamma$ for $\gamma \to \infty$ [10]. We conclude that the γ dependence of G_s for large γ is the same as for $G_s(0, 0)$; however, the prefactors are different.

The collective modes are given as the poles of $X_s(q, \omega)$. One finds in the long wavelength limit $\omega_s(q)/v_F|q| = [\kappa_0/\kappa_s]^{1/2}$ [16]. For the sound velocity v_s of spin (s) waves, using $\omega_s(q \to 0) = v_s|q|$, we obtain

$$v_s = v_F/(1 + 2\gamma/\pi^2)^{1/2}. \tag{15}$$

The relation for spin waves is analogous to the relation found for the sound velocity v_d of density (d) waves given by $v_d = v_F[\kappa_0/\kappa]^{1/2}$ with $\omega_d(q \to 0) = v_d|q|$ and κ as the compressibility of the system [16]. Analytical results for κ_0/κ have been given in [5].

Numerical results for v_d/v_F and v_s/v_F versus γ are shown in figure 4 together with the results of the STLS approach. The STLS results for the compressibility and the spin susceptibility are obtained from GSE calculations [15]. We conclude that the STLS approach is trustworthy for $\gamma < 10 \approx 2\gamma_{HFA}$. Within the LST approach [10] one finds $v_s/v_F = 0.5$ for $\gamma \to \infty$ because $G_s(0, 0) = 2G_s$.

In the literature the difference between v_d and v_s is called spin-charge separation [19]. Numerical results similar to our results as shown in figure 4 have been found for the one-dimensional Hubbard model [20]. The two parameters K_d and K_s, defined by $\partial^2 E_g/\partial N^2 = \pi v_d/2K_d$ and by $\partial^2 E_g/N^2 \partial \xi^2 = \pi v_s/2K_s$, are related to non-universal

exponents of the power-law behaviour of correlation functions [19]. For our model we find $K_d = (\kappa/\kappa_0)^{1/2}$ [5] and $K_s = (\kappa_s/\kappa_0)^{1/2} = (1 + 2\gamma/\pi^2)^{1/2}$. We conclude that analytical results are obtained for K_d and K_s. We are not aware that in the literature analytical results have been obtained for K_s.

6. Discussion

6.1. The polarization ansatz

The essential approximation in this paper was the approximation for polarization dependence of the correlation energy. We used $f(\xi) = 1-\xi^2$. We have chosen this form for convenience and because the application of three different methods leads to the same form of the spin susceptibility.

In fact, we could introduce a more complicated function, for instance $f(\xi) = 1-p(\gamma)\xi^2$ with $p(\gamma \to \infty) = 1 + \mathrm{O}(1/\gamma^2)$ and $p(\gamma \to 0) = 3/2$. For the spin susceptibility we obtain

$$\frac{\kappa_0}{\kappa_s} = \frac{1 - 4\gamma^2[1 - p(\gamma)]/\pi^4}{1 + 2\gamma/\pi^2}. \tag{16}$$

With $p(\gamma) = (3 + \gamma^2)/(2 + \gamma^2)$, which has the correct asymptotical behaviour, we derive in the weak coupling limit $\kappa_0/\kappa_s = 1 - 2\gamma/\pi^2 + 6\gamma^2/\pi^4$. This is the exact expression for weak coupling. In the strong coupling limit, using $p(\gamma \to \infty) = 1 + 1/\gamma^2$, we find $\kappa_0/\kappa_s = (1 + 4/\pi^4)/(1 + 2\gamma/\pi^2)$ and the factor $4/\pi^4 \approx 0.04$ in this expression is due to the use of the function $p(\gamma)$. This shows that in the strong coupling limit $p(\gamma)$ modifies our results by less than 4%. We have verified numerically that for $\gamma < 10$ the corrections introduced by $p(\gamma)$ are even smaller than 4%. This is clear because our results obtained before with $p(\gamma) = 1$ ($f(\xi) = 1 - \xi^2$) are in agreement with the STLS approach, which is the correct weak coupling theory, see figure 3. We believe that these arguments are convincing and justify $f(\xi) = 1-\xi^2$. However, the discussion also shows that even 'more accurate' results can be obtained by using a 'more sophisticated' polarization dependence of the correlation energy.

We would like to mention that for the LFC, including $p(\gamma)$, we obtain

$$G_s(0,\ 0) = \frac{1 + 2\gamma[1 - p(\gamma)]/\pi^2}{2(1 + 2\gamma/\pi^2)} \tag{17}$$

and $p(\gamma \to \infty)$ does not modify the strong coupling limit.

6.2. Results

The collective modes are determined by the compressibility and the spin susceptibility. Our analytical results concerning spin-density modes and the spin susceptibility might be useful for experimenters [21]. We mention that our theory describes collective density modes and collective spin modes together with one-particle (electron–hole) excitations, see equation (12). In experiments with quantum wires these three kinds of mode have been observed [22–24]. Note that within theories using the bosonization approach one-particle excitations do not exist [19, 20]—they become lost in the bosonization approximation. Of course, in order to describe qualitatively the collective modes found in experiments with quantum wires [22–24], a long-range Coulomb interaction potential [25] should be used in the calculation. However, we believe that the model discussed in this paper is the simplest model in solid-state physics in order to study many-body effects.

From our results for the spin susceptibility, the collective modes and the comparison with results obtained from GSE calculations within the STLS approach we conclude that the STLS approach gives quantitatively accurate results for $\gamma < 2\gamma_{HFA} \approx 10$. We believe that the case of a short-range interaction potential is the worst case for the application of the STLS approach. The reason is that the GSE within the STLS approach becomes completely wrong for large coupling, $\varepsilon_0(\gamma, \xi = 0) \propto \ln(\gamma)$ [3, 10], and this is not the case for long-range interaction potentials [26].

7. Conclusion

We presented analytical results for the ground-state energy and the spin susceptibility of a one-dimensional electron gas with a short-range interaction potential as a function of the coupling parameter γ and the spin-polarization parameter ξ. These results are quasi-exact. We calculated collective modes for density and spin excitations.

The parameter $\gamma_{HFA} = \pi^2/2$ was identified as an essential value of the coupling parameter γ. The validity range of the STLS approach was discussed and we found that for $\gamma < 2\gamma_{HFA} \approx 10$ this approach is accurate.

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